Implicit Differentiation.

Up to this point, we have been working with functions that have been expressed *explicitly*. That is to say, they have been written in the form $y = f(x)$.

-examples-

Some equations involving x and y, though, cannot easily be expressed explicitly (cannot easily be solved for y). In fact, some cannot be solved for y at all! In these equations, we say that x and y are related ***implicitly***.

-examples-

In order to find $\frac{dy}{dx}$ in *implicit* relations, we use a process called ***implicit differentiation.*** In this process, we treat y as though it were some unknown "function" of x, and thus apply chain rule whenever we differentiate terms involving y.

Recall chain rule: $\frac{d}{dx}\left[f(u)\right] =$

-examples- a. $\frac{d}{dx}\left(x^3\right)$ b. $\frac{d}{dx}\left(y^3\right)$

-example- Find $\frac{dy}{dx}$: $x^2 - 4y^2 = 2$

-example- Find $\frac{dy}{dx}$: $4x^2y^3 + 5x - 2y = -4$. Then find the equation of the line tangent to the curve at the point (2, −1).

-example- Find $\frac{dy}{dx}$: $x^2 + \sin(2x + 3y) - 4y = 6$

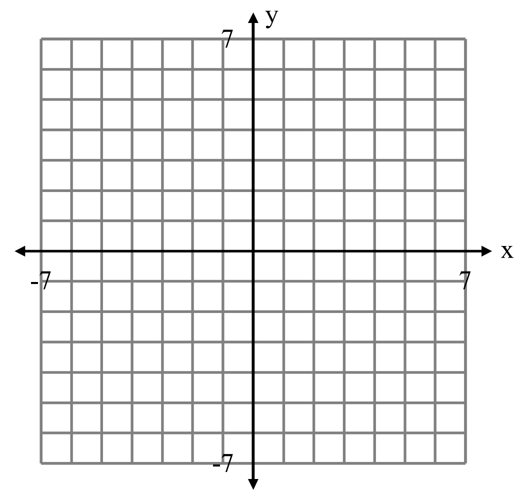
-example- Given $x^2 - 3y^3 = 8$, find $\dfrac{d^2y}{dx^2}$.

-example- Consider the curve given by $y + \cos y = x + 1$ for $0 \le y \le 2\pi$

a. Find $\dfrac{dy}{dx}$

b. Determine where the curve has a vertical tangent line, then write an equation for each vertical tangent to the curve.

-example- *Relating* $\frac{dy}{dx}$ *values to the graph of the relation.*



a. Sketch the graph of $x^2 + y^2 = 25$

b. Find $\frac{dy}{dx}$, and use it to determine the equations of the lines tangent to the curve at the points (3, 4) and (–3, 4). Draw these tangent lines on your graph.

c. From your graph, determine where the curve has horizontal tangent lines, and where the curve has vertical tangent lines.

Verify the slopes of these lines using your expression for $\frac{dy}{dx}$.